Nonlinear regimes of the resonant acoustic transparency for longitudinal–transverse elastic waves in low-temperature paramagnetic crystals

2004 J. Phys.: Condens. Matter 16 1733

(http://iopscience.iop.org/0953-8984/16/10/006)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **16** (2004) 1733–1749 PII: S0953-8984(04)67574-6

Nonlinear regimes of the resonant acoustic transparency for longitudinal–transverse elastic waves in low-temperature paramagnetic crystals

A V Gulakov and S V Sazonov

Physics Department, Kaliningrad State University, 14 Nevskogo Street, Kaliningrad 236041, Russia

E-mail: nst@alg.kaliningrad.ru

Received 12 August 2003, in final form 11 February 2004
Published 27 February 2004
Online at stacks.iop.org/JPhysCM/16/1733 (DOI: 10.1088/0953-8984/16/10/006)

Abstract

Nonlinear propagation of longitudinal–transverse acoustic pulses in the system of resonant paramagnetic impurities is theoretically investigated. It is shown that, under a large transverse pulse component and propagation along an external magnetic field, a regime of acoustic self-induced transparency is realized. A new soliton mode, following by reduction in the propagation velocity, similar to the case of self-induced transparency is realized in the opposite limit. Then, however, populations of the quantum Zeeman sublevels stay practically unchanged and the transverse component of the pulse suffers phase modulation.

1. Introduction

Analysis of the development of coherent nonstationary phenomena in physics shows that, besides other effects, corresponding optical phenomena have found their acoustic analogues at the same time. Such a situation is the case for self-induced transparency (SIT) [1] and its acoustic analogue (ASIT) [2–4]. One of the main differences for acoustic waves in solids from the optical case is that acoustic waves have a longitudinal–transverse structure. Acoustic self-induced transparency (ASIT) for longitudinal–transverse elastic waves in a paramagnetic crystal on the spin system $S = 1/2$ was studied in detail in the following work [5, 6]. During the propagation of pulses along the external magnetic field **B** two types of acoustic solitons were revealed. They correspond to two opposite limiting cases, when the amplitude of one of the components (transverse or longitudinal) is much larger than the other. The simple ASIT regime is realized in the first case, when the propagation of the 2π pulse is accompanied by population inversion of the Zeeman sublevels with the subsequent return of the medium to the initial state. If the transverse component dominates the soliton mode is realized, during which the populations of the Zeeman sublevels practically do not change. Note that, in both cases, pulses experience practically the same slowing down in their propagation velocities [5].

It is shown [6, 7] that the description of the propagation of resonant longitudinal–transverse elastic pulses in a paramagnetic cubic crystal can be, under some assumptions, restricted to systems of material and wave equations, which can be integrated by the inverse scattering technique (IST). Analysis was carried out with the use of the slowly varying envelope approximation (SVEA) for the transverse component and without any given approximation. In the last case an approximation of the low density of paramagnetic centres was made. The longitudinal component of the pulse in both cases has no carrier frequency, i.e. it was a video pulse [5–7].

As was said above, paramagnetic impurities with effective spin $S = 1/2$ were considered as the objects of interaction with acoustic pulses in solids [5–8]. At the same time it is well known that paramagnetic centres with effective spin $S = 1$ feel the strongest interaction with oscillations of the lattice [9]. Ions of Fe^{2+} and Ni^{2+} in a crystal matrix of MgO can be taken as examples of the latter [2]. So, from the point of view of the experimental test of the theoretical conclusion it seems appropriate to investigate ASIT for longitudinal–transverse elastic pulses, propagating in a system of paramagnetic centres with effective spin $S = 1$. Thus, the present paper is devoted to researching this problem.

The paper is organized as follows. In section 2, on the basis of the semi-classical Hamilton formalism the system of material and wave equations, describing the propagation of longitudinal–transverse hypersound in a system of paramagnetic impurities with effective spin $S = 1$ is derived. For the transverse component, exciting the quantum transitions inside a Zeeman triplet, SVEA is used. Contrary to the longitudinal hypersound component, producing a dynamical shift of the quantum sublevels has no carrier frequency. For the given component in view of the small concentration of paramagnetic impurities the approximation of unidirectional propagation is used. This allows us to reduce the corresponding wave equation to first order with respect to derivatives. In section 3 the material equations for the density matrix are solved with the use of the asymptotical Wentzel–Brillouin–Kramers–Jeffry (WBKJ) method. As a result the material variables are expressed through the wave variables and the investigation is reduced to finding the solutions of the nonlinear integro-differential wave equations for transverse and longitudinal components of the acoustic field. Corresponding analytical solutions in the opposite limits of domination of the transverse and longitudinal components of the pulse are studied in section 4. In the first case the ASIT mode is realized, while in the second limit we have the soliton mode. This is called by us the acoustic longitudinal–transverse transparency (ALTT). The intermediate case is investigated by means of numerical simulations for pulses, propagating in the stationary regime. In the conclusion (section 5) we review the results obtained here and indicate the future prospects for investigation.

2. Semiclassical self-consistent equations of motion

For the description of the interaction of the elastic field with paramagnetic centres we will use the semi-classical approach, according to which the field will be described by classical equations of the continuous medium and paramagnetic impurities by quantum mechanics equations.

Let the external magnetic field **B** be oriented along the z axis which coincides with the fourth-order axis of the cubic crystal, which contains paramagnetic impurities. Then the Hamiltonian of some spin in the given field, which also interacts with oscillations of the lattice, can be written in the following form:

$$\hat{H} = \hat{H}_S + \hat{V}, \tag{1}$$

where its own Hamiltonian of spin is

$$\hat{H}_S = g\mu_B B\hat{S}_z. \tag{2}$$

Here g is the Landé factor, μ_B is the Bohr magneton and the Hamiltonian $\hat{V}$ of the spin–phonon interaction in its most common form is a function of the bilinear combination of spin operators $\hat{S}_j$ $(j = x, y, z)$ [9]:

$$\hat{V} = f(\hat{S}_i Q_{ij} \hat{S}_j). \tag{3}$$

Spin matrices in the accepted geometry look like [20]

$$\hat{S}_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \hat{S}_y = \frac{\mathrm{i}}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$\hat{S}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{4}$$

Elements Q_{ij} of the matrix $\hat{Q}$ depend on the components of the strain tensor

$$\mathcal{E}_{ml} = \frac{1}{2}\left(\frac{\partial U_m}{\partial x_l} + \frac{\partial U_l}{\partial x_m}\right), \tag{5}$$

where $m, l = x, y, z$; U_m are the components of the displacement vector $\mathbf{U}$ of the continuous medium; in (3) and hereafter iterated indexes mean summation.

In the absence of deformation $\hat{Q}$ is an unitary matrix that is $Q_{ij} = \delta_{ij}$. Expanding $\hat{Q}$ in (3) and then f in the series by $\mathcal{E}_{ml}$ and limiting the linear terms, we will derive [10]

$$f = f(\hat{\mathbf{S}}^2) + f'(\hat{\mathbf{S}}^2)\left(\frac{\partial Q_{ij}}{\partial \mathcal{E}_{ml}}\right)_0 \hat{S}_i \hat{S}_j \mathcal{E}_{ml},$$

where the subscript '0' means the derivative under the condition that $\mathcal{E}_{ml} = 0$. $f(\hat{\mathbf{S}}^2)$ here can be rejected as an additive constant component to the spin Hamiltonian, dependent on the Kazimir operator $\hat{\mathbf{S}}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2 = S(S+1) = 2$. Then the Hamiltonian of the spin–phonon coupling in the concerned approximation take the form [9, 10]

$$\hat{V} = G_{ijml}\mathcal{E}_{ml}\hat{S}_i\hat{S}_j = \tfrac{1}{2}G_{ijml}\mathcal{E}_{ml}(\hat{S}_i\hat{S}_j + \hat{S}_j\hat{S}_i), \tag{6}$$

where the components of the tensor $\hat{G}$ of the spin–phonon interaction are $G_{ijml} = f'(\hat{\mathbf{S}}^2)(\partial Q_{ij}/\partial \mathcal{E}_{ml})_0$.

From the definition of $\hat{G}$ and (3) one can see that its components are symmetric relative to the transposition of index pairs i, j and m, l and also relative to the transposition inside these pairs. This property was used in (6).

The physical mechanism of spin–phonon interaction in the concerned case is the so-called Van Vleck mechanism [9, 11, 12], according to which the elastic wave modulates the intracrystalline electric field in place of the disposition of paramagnetic ions. Gradients of the given field then cause transitions between the Zeeman sublevels of ions.

Note that for spin $S = 1/2$ we have $\hat{S}_i\hat{S}_j + \hat{S}_j\hat{S}_i = 0$ because of the anticommutative property of the Pauli matrix. Consequently, according to (6), $\hat{V} = 0$. In this case the spin–phonon interaction is conditioned by modulation of the Landé tensor components by the elastic field, and so the Hamiltonian of the spin–phonon coupling has become nonlinear by spin operators [3, 8, 12, 13]. For the spin $S = 1$ the quadratic spin operator interaction has become two orders of magnitude stronger than the linear [9, 11]. Because of this we neglect the latter here.

For the self-consistent description of the dynamics of spin and acoustic pulses let us add the Hamiltonian of the elastic field to (2) and (6). As the given field is classical, so its Hamiltonian in the approximation of a continuous medium has the form of a classical function [14]:

$$H_{\mathrm{a}} = \frac{1}{2}\int\left(\frac{p_i p_i}{\rho} + C_{ijkl}\frac{\partial U_i}{\partial x_j}\frac{\partial U_k}{\partial x_l}\right)\mathrm{d}^3\mathbf{r}, \tag{7}$$

where ρ is the average density of the crystal, p_i ($i = x, y, z$) are the components of the impulse density $\mathbf{p}$ of the local displacement of the medium and C_{ijkl} are the components of the elastic modulus tensor of the medium.

There are three independent elastic constants in the case of the cubic crystal: $C_{xxxx} \equiv C_{11}$, $C_{xxyy} \equiv C_{12}$ and $C_{xyxy} \equiv C_{44}$ (here the index Voight notation is used [9, 11]: $xx \to 1$, $yy \to 2$, $zz \to 3$, $yz \to 4$, $xz \to 5$, $xy \to 6$). The expression (7) in that case takes the form [14]

$$H_{\mathrm{a}} = \frac{1}{2}\int\left[\frac{p_x^2 + p_y^2 + p_z^2}{\rho} + C_{11}(\mathcal{E}_{xx}^2 + \mathcal{E}_{yy}^2 + \mathcal{E}_{zz}^2)\right.$$
$$\left. + 2C_{12}(\mathcal{E}_{xx}\mathcal{E}_{yy} + \mathcal{E}_{xx}\mathcal{E}_{zz} + \mathcal{E}_{yy}\mathcal{E}_{zz}) + 4C_{44}(\mathcal{E}_{xy}^2 + \mathcal{E}_{xz}^2 + \mathcal{E}_{yz}^2)\right]\mathrm{d}^3\mathbf{r}. \tag{8}$$

Here the axes x, y and z coincide with the main axes of the fourth-order cubic crystal.

According to the general scheme of the semi-classical approach [5, 8, 10], spin evolution will be described by the equation for the density matrix:

$$\mathrm{i}\hbar\frac{\partial\hat{\rho}}{\partial t} = [\hat{H}_S + \hat{V}, \hat{\rho}], \tag{9}$$

where $\hbar$ is the Planck constant, and the dynamics of the acoustic field will be described by the classical Hamilton equation for the continuous medium:

$$\frac{\partial\mathbf{p}}{\partial t} = -\frac{\delta}{\delta\mathbf{U}}(H_{\mathrm{a}} + \langle\hat{\hat{V}}\rangle), \qquad \frac{\partial\mathbf{U}}{\partial t} = \frac{\delta}{\delta\mathbf{p}}(H_{\mathrm{a}} + \langle\hat{\hat{V}}\rangle). \tag{10}$$

Here $\langle\hat{\hat{V}}\rangle = \int n\langle\hat{V}\rangle\,\mathrm{d}^3\mathbf{r}$, n is the concentration of the paramagnetic centres and $\langle\hat{V}\rangle \equiv Sp(\hat{\rho}\hat{V})$ is the quantum average by the spin density matrix. In (9) we neglected the relaxation terms considering that the duration of the acoustic pulse is shorter than all the relaxation times. Hereafter we will also neglect inhomogeneous broadening.

The system of equations (9) and (10) with the use of (2), (6) and (7) allows us to describe propagation of the acoustic pulse in any given direction with respect to $\mathbf{B}$. Below we assume that conditions of the Faraday geometry are realized, which means that the pulse propagates along $\mathbf{B}$ (along the z axis). We will assume correspondingly that all dynamical variables depend only on z and t. However, these variables are not dependent on x and y. Then three non-zero components of the strain tensor remain: $\mathcal{E}_{zz} = \mathcal{E}_{\|} = \partial U_z/\partial z$, $\mathcal{E}_{xz} = 0.5\partial U_x/\partial z$, $\mathcal{E}_{yz} = 0.5\partial U_y/\partial z$, $\mathcal{E}_{xx} = \mathcal{E}_{yy} = \mathcal{E}_{xy} = 0$. In this case one can rewrite the expression (8) for H_{a} in the form

$$H_{\mathrm{a}} = \frac{1}{2}\int\left\{\frac{p_x^2 + p_y^2 + p_z^2}{\rho} + C_{11}\left(\frac{\partial U_z}{\partial z}\right)^2 + C_{44}\left[\left(\frac{\partial U_x}{\partial z}\right)^2 + \left(\frac{\partial U_y}{\partial z}\right)^2\right]\right\}\mathrm{d}^3\mathbf{r}. \tag{11}$$

For components of the tensor $\hat{G}$ we will use below the Voight notation [9, 11]. In view of the cubic symmetry $G_{23} = G_{13}$, $G_{33} = G_{22} = G_{11}$, $G_{55} = G_{44}$. In an effort to further simplification of the expression for $\hat{V}$ in a cubic crystal we note that, during inversion of the coordinate axes x and y, the components of the spin operators change in the following way [10]: $x \to -x$, $\hat{S}_x \to \hat{S}_x$, $\hat{S}_y \to -\hat{S}_y$, $\hat{S}_z \to -\hat{S}_z$; $y \to -y$, $\hat{S}_x \to -\hat{S}_x$, $\hat{S}_y \to \hat{S}_y$,

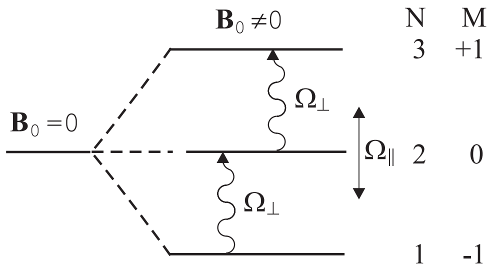

Figure 1. Scheme of the quantum transition during Zeeman splitting in the three-level system. Here N is the number of the quantum level, M is a magnetic quantum number, the wavy arrow is the quantum transition induced by the transverse component of the acoustic field and $\leftrightarrow$ is a dynamical chirp of the middle quantum level.

$\hat{S}_z \to -\hat{S}_z$. Taking into consideration the invariance of $\hat{V}$ relative to the given operations, we write

$$\hat{V} = \frac{3}{2}G_{11}\hat{S}_z^2\frac{\partial U_z}{\partial z} + \frac{1}{2}G_{44}\left[\frac{\partial U_x}{\partial z}(\hat{S}_z\hat{S}_x + \hat{S}_x\hat{S}_z) + \frac{\partial U_y}{\partial z}(\hat{S}_z\hat{S}_y + \hat{S}_y\hat{S}_z)\right]. \quad (12)$$

By summing all the above, after the use of (7), (10) and (12) we will have

$$\frac{\partial^2\mathcal{E}_\perp}{\partial t^2} - a_\perp^2\frac{\partial^2\mathcal{E}_\perp}{\partial z^2} = \frac{n}{2\rho}G_{44}\frac{\partial^2}{\partial z^2}(\rho_{32}^* - \rho_{21}^*), \quad (13)$$

$$\frac{\partial^2\mathcal{E}_\parallel}{\partial t^2} - a_\parallel^2\frac{\partial^2\mathcal{E}_\parallel}{\partial z^2} = -\frac{3n}{2\rho}G_{11}\frac{\partial^2\rho_{22}}{\partial z^2}, \quad (14)$$

where the complex transverse strain is $\mathcal{E}_\perp \equiv (\mathcal{E}_{xz} + \mathrm{i}\mathcal{E}_{yz})/\sqrt{2}$ and $a_\perp = \sqrt{C_{44}/\rho}$ and $a_\parallel = \sqrt{C_{11}/\rho}$ are, respectively, the velocities of the transverse and longitudinal sound in the absence of paramagnetic impurities.

Using (2), (4) and (6), one can rewrite the expression for the operator $\hat{H}_\mathrm{a} + \hat{V}$, appearing in (9) in the matrix form

$$\hat{H}_S + \hat{V} = \begin{pmatrix} \hbar\omega_0 + \frac{3}{2}G_{11}\mathcal{E}_\parallel & \frac{G_{44}}{2}\mathcal{E}_\perp^* & 0 \\ \frac{G_{44}}{2}\mathcal{E}_\perp & 0 & -\frac{G_{44}}{2}\mathcal{E}_\perp^* \\ 0 & -\frac{G_{44}}{2}\mathcal{E}_\perp & -\hbar\omega_0 + \frac{3}{2}G_{11}\mathcal{E}_\parallel \end{pmatrix}, \quad (15)$$

where $\omega_0 = g\mu_\mathrm{B}B/\hbar$ is a frequency of the Zeeman splitting in the equidistant three-level system with spin $S = 1$ (see figure 1). Note that numbering of the quantum levels goes from the bottom to the top.

From (13)–(15) one can see that, in the Faraday geometry, the transverse component of the acoustic field induces cascade quantum transitions $1 \leftrightarrow 2$ and $2 \leftrightarrow 3$. At the same time the transverse component displaces the middle (with number 2) quantum level. Due to this, the frequencies of the named transitions suffer a dynamical shift (see figure 1).

Further we will consider that the quasi-monochromatic circularly polarized transverse component of the elastic pulse has a carrier frequency ω_0 and wavenumber $k = \omega_0/a_\perp$. Thus we can write

$$\frac{G_{44}}{2\hbar}\mathcal{E}_\perp = \Omega_\perp \mathrm{e}^{\mathrm{i}(\omega_0 t - kz)}, \quad (16)$$

where $\Omega_\perp$ is the slowly varying envelope (with the dimension of frequency), satisfying the condition

$$\left|\frac{\partial\Omega_\perp}{\partial t}\right| \ll \omega_0|\Omega_\perp|, \qquad \left|\frac{\partial\Omega_\perp}{\partial z}\right| \ll k|\Omega_\perp|.$$

Non-diagonal components of $\hat{\rho}$ will be given by

$$\rho_{32} = R_{32}\mathrm{e}^{-\mathrm{i}(\omega_0 t - kz)}, \qquad \rho_{21} = R_{21}\mathrm{e}^{-\mathrm{i}(\omega_0 t - kz)}, \\ \rho_{31} = R_{31}\mathrm{e}^{-2\mathrm{i}(\omega_0 t - kz)}, \tag{17}$$

where R_{32}, R_{21} and R_{31} are the slowly varying envelopes with the same meaning as $\Omega_\perp$.

In compliance with SVEA during the substitution of (16) and (17) into (13) we will neglect the second-order derivatives of $\Omega_\perp$ and both derivatives of ρ_{32}^* and ρ_{21}. Substitution of (16) and (17) into (9) taking into consideration (15) and with the preservation of all items bring us to the equation for the substantial matrix $\hat{R}$ with dimensions of 3×3, in which diagonal components are given by ρ_{33}, ρ_{22} and ρ_{11} and non-diagonal by $R_{32} = R_{23}^*$, $R_{31} = R_{13}^*$ and $R_{21} = R_{12}^*$.

The right-hand side of (14) contains the slow (in comparison with the fast oscillation with frequency ω_0 of the non-diagonal elements $\hat{\rho}$) variable ρ_{22}. Hence, the transverse component of the displacement $\mathcal{E}_\parallel$ has no carrier frequency. Therefore we cannot apply SVEA to (14). However, the reduction of the given equation with respect to derivatives can be performed, using the other approximation, namely the approximation of the lower density of resonant paramagnetic impurities. Meanwhile we note that the right-hand side of (14), proportional to n, will be considered as a small perturbation. Also we note that such an approach, without using SVEA, for the optical problems was first suggested in [15, 16]. In a zero approach by n for the solution of (14) we have two waves, travelling in opposite directions with velocity $a_\parallel$. We neglect the wave travelling opposite to the z axis and will be interested only in the wave propagating along the given axis. This approximation must be carried out well in the paramagnetic sample, giving a small mistake in the input. Then in the first approach we write the solution of (14) in the form $\mathcal{E}_\parallel = \mathcal{E}_\parallel(\tau, \zeta)$, where the local time $\tau = t - z/a_\parallel$, the 'slow' coordinate $\zeta = \mu z$ and μ is a small ($\mu \ll 1$) nondimensional parameter proportional to n. It is obvious that

$$\frac{\partial^2 \mathcal{E}_\parallel}{\partial t^2} = \frac{\partial^2 \mathcal{E}_\parallel}{\partial \tau^2}, \qquad \frac{\partial \mathcal{E}_\parallel}{\partial z} = -\frac{1}{a_\parallel}\frac{\partial \mathcal{E}_\parallel}{\partial \tau} + \mu \frac{\partial \mathcal{E}_\parallel}{\partial \zeta}.$$

Neglecting the term $\sim\mu^2$, we write

$$\frac{\partial^2 \mathcal{E}_\parallel}{\partial z^2} \approx \frac{1}{a_\parallel^2}\frac{\partial^2 \mathcal{E}_\parallel}{\partial \tau^2} - \frac{2\mu}{a_\parallel}\frac{\partial^2 \mathcal{E}_\parallel}{\partial \tau\, \partial \zeta}.$$

Substituting the given expression for the derivatives into (14), after integration with respect to τ and taking into account the zero values of the deformation and its derivatives at infinity we obtain the first-order wave equation for $\mathcal{E}_\parallel$.

By summing all expressions after (16), we rewrite the system of wave and substantial equations in the form of

$$\frac{\partial \Omega_\perp}{\partial z} + \left(\frac{1}{a_\perp} - \frac{1}{a_\parallel}\right)\frac{\partial \Omega_\perp}{\partial \tau} = \mathrm{i}\beta_\perp(R_{32}^* - R_{21}^*), \tag{18}$$

$$\frac{\partial \Omega_\parallel}{\partial z} = -\beta_\parallel \frac{\partial \rho_{22}}{\partial \tau}, \tag{19}$$

$$\frac{\partial \hat{R}}{\partial \tau} = -\mathrm{i}[\hat{\Omega}, \hat{R}], \tag{20}$$

where

$$\hat{R} = \begin{pmatrix} \rho_{33} & R_{32} & R_{31} \\ R_{32}^* & \rho_{22} & R_{21} \\ R_{31}^* & R_{21}^* & \rho_{11} \end{pmatrix},$$

$$\hat{\Omega} = \begin{pmatrix} \Omega_\parallel & \Omega_\perp^*/2\sqrt{2} & 0 \\ \Omega_\perp/2\sqrt{2} & 0 & -\Omega_\perp^*/2\sqrt{2} \\ 0 & -\Omega_\perp/2\sqrt{2} & \Omega_\parallel \end{pmatrix}, \tag{21}$$

$$\Omega_\parallel = 3G_{11}\mathcal{E}_\parallel/2\hbar, \qquad \beta_\perp = n\omega_0 G_{44}^2/(2\sqrt{2}\hbar\rho a_\perp^3),$$

$$\beta_\parallel = 9nG_{11}^2/(8\hbar\rho a_\parallel^3).$$

We emphasize one more that wave equations for the transverse and longitudinal components of the acoustic pulse are reduced from the initial second-order system (13) and (14) to the first-order system (18) and (19) by different physical causes and with the use of different mathematical approximations. In (18) the complex quantity $\Omega_\perp$ has a meaning of a slowly varying envelope of the quasi-monochrome transverse pulse component with a carrier resonant frequency ω. At the same time equation (19) is written not for the envelope but directly for the longitudinal component $\Omega_\parallel$, which can have no carrier frequency at all. The equation for $\Omega_\parallel$ reduces to the form (19) because of the assumption about the low concentration of resonant paramagnetic impurities.

From the derivation of (19) it is clear that the profile of the longitudinal component slowly ($\sim\mu$) varies in the travelling framework, moving with a velocity $a_\parallel$. For this reason that approximation is sometimes called the slowly varying profile approximation (SVPA) [17], which it is important not to confuse with the SVEA.

3. Nonlinear wave equations

Let us exempt material variables from (18)–(21) by the use of the operator variant of the asymptotic Wentzel–Brillouin–Kramers–Jeffry (WBKJ) method [18, 19]. From (21) one can see that the matrix $\hat{\Omega}$ does not commute with itself at different moments in time. However, if the impulse excitation is sufficiently short the fluctuation of $\hat{\Omega}$ during this excitation time $\Delta\tau$ is insignificant and one can talk about the approximate commutativity. Then [5]

$$\hat{R}(\tau) = \hat{U}\hat{R}(t_0)\hat{U}^+, \tag{22}$$

where the evolution operator is

$$\hat{U} = \lim_{\Delta\tau\to 0}[\exp(-\mathrm{i}\hat{\theta})], \tag{23}$$

where $\hat{\theta} = \int_{t_0}^{t_0+\Delta\tau} \hat{\Omega}\,\mathrm{d}\tau'$ is the area operator and t_0 is the starting time of the impulse excitation.

With the decrease in the pulse temporal duration its amplitude rises and the system (20) and (21) takes a formal look of the linear equation with large variable coefficients. Given these circumstance let us here talk about the application of WBKJ [18, 19].

The operator exponent can be calculated with the use of the Silvester formula [20]:

$$\exp(-\mathrm{i}\hat{\theta}) = \sum_j \prod_{q\neq j} \frac{\hat{\theta} - \lambda_q \hat{I}}{\lambda_j - \lambda_q} \exp(-\mathrm{i}\lambda_j), \tag{24}$$

where $\hat{I}$ is the identity matrix and $\{\lambda_q\}$ are the set of eigenvalues of $\hat{\theta}$.

We reveal the indeterminacy of the type 0/0 in the multipliers behind the exponents by L'Hôpital's rule, also assuming that in the limit $\lambda_j \approx p_j\Delta\tau \approx \int_{t_0}^{t_0+\Delta\tau} p_j\,\mathrm{d}\tau$, where $\{p_j\}$ is the spectrum of the eigenvalues of the matrix $\hat{\Omega}$.

Then, using (23) and (24), we obtain

$$\hat{U} = \sum_j \prod_{q\neq j} \frac{\hat{\Omega} - p_q \hat{I}}{p_j - p_q} \exp\left(-\mathrm{i}\int_{-\infty}^{\tau} p_j \, \mathrm{d}\tau'\right). \tag{25}$$

Here time t_0 formally tends to $-\infty$.

From (21) without difficulty we find

$$p_1 = \Omega_\parallel, \qquad p_{2,3} = (\Omega_\parallel \pm \Omega)/2, \qquad \Omega = \sqrt{\Omega_\parallel^2 + |\Omega_\perp|^2}. \tag{26}$$

Considering that before the impulse excitation

$$\hat{R}(t_0) = \hat{R}(-\infty) = \begin{pmatrix} W_3 & 0 & 0 \\ 0 & W_2 & 0 \\ 0 & 0 & W_1 \end{pmatrix},$$

where W_j ($j = 1, 2, 3$) are the initial populations of the quantum Zeeman sublevels, satisfying the condition $W_1 + W_2 + W_3 = 1$, from (22), (25), (26) and (21) we will have

$$R_{32}^* - R_{21}^* = \frac{\Omega_\perp}{\sqrt{2}\Omega}\left[(1-3W_2)\frac{\Omega_\parallel}{\Omega}\sin\frac{\theta}{2} + \mathrm{i}(W_1 - W_3)\cos\frac{\theta_\parallel}{2}\right]\sin\frac{\theta}{2}, \tag{27}$$

$$\rho_{22} = W_2 + \frac{1-3W_2}{2}\left(\frac{|\Omega_\perp|}{\Omega}\right)^2 \sin^2\frac{\theta}{2}. \tag{28}$$

Here $\theta_\parallel = \int_{-\infty}^{\tau} \Omega_\parallel \, \mathrm{d}\tau'$, $\theta = \int_{-\infty}^{\tau} \Omega \, \mathrm{d}\tau'$. The common property of the approximate solutions, derived with the help of the WBKJ method, is that coefficients of a periodic function vary slowly. Then the given functions change over time [18, 19]:

$$\begin{aligned}\frac{\partial\rho_{22}}{\partial\tau} &= \frac{1-3W_2}{2}\frac{\partial}{\partial\tau}\left(\frac{|\Omega_\perp|}{\Omega}\right)^2\sin^2\frac{\theta}{2} \\ &\approx \frac{1-3W_2}{2}\left(\frac{|\Omega_\perp|}{\Omega}\right)^2\frac{\partial}{\partial\tau}\left(\sin^2\frac{\theta}{2}\right) = \frac{1-3W_2}{2}\frac{|\Omega_\perp|^2}{\Omega}\sin\theta.\end{aligned}$$

From here, and also from (18), (19) and (27) after the representation $\Omega_\perp = |\Omega_\perp|\mathrm{e}^{\mathrm{i}\varphi}$ we will derive a system of nonlinear integro-differential wave equations:

$$\frac{\partial|\Omega_\perp|}{\partial z} + \left(\frac{1}{a_\perp} - \frac{1}{a_\parallel}\right)\frac{\partial|\Omega_\perp|}{\partial\tau} = -\alpha_\perp\frac{|\Omega_\perp|}{\Omega}\cos\frac{\theta_\parallel}{2}\sin\frac{\theta}{2}, \tag{29}$$

$$\frac{\partial\varphi}{\partial z} + \left(\frac{1}{a_\perp} - \frac{1}{a_\parallel}\right)\frac{\partial\varphi}{\partial\tau} = \sigma_\perp\frac{\Omega_\parallel}{\Omega^2}\sin^2\frac{\theta}{2}, \tag{30}$$

$$\frac{\partial\Omega_\parallel}{\partial z} = -\alpha_\parallel\frac{|\Omega_\perp|^2}{\Omega}\sin\theta, \tag{31}$$

where $\alpha_\perp = \beta_\perp(W_1 - W_3)/\sqrt{2}$, $\sigma_\perp = \beta_\perp(1-3W_2)/\sqrt{2}$ and $\alpha_\parallel = \beta_\parallel(1-3W_2)/4$. The system of (29)–(31) describes nonlinear interactions between long wave longitudinal and short wave transverse elastic components by resonant paramagnetic impurities. From (31) one can see that the transverse component can generate the longitudinal component. The presence of the latter drives the phase modulation of the transverse component. Obviously, the interaction between both components will be most effective when $a_\perp = a_\parallel$. The given equality itself expresses the condition of long–short wave resonance of Zakharov–Benney [21]. If the short wave component does not stay in resonance with the atomic subsystem, the latter is excited slightly. The regime of long–short wave resonance in this case is described by the system of Zakharov having weak power nonlinearity [22]. The unidirectional variant of the Zakharov system is the integrated system of Yajima–Oikawa [21–23]. In our case a strong interaction of the field with the medium takes place, generally speaking, because of the transverse component and spin subsystem resonance. So, nonlinear systems (29)–(31) have a pronounced nonpower nature.

The following is based on analysis of the system of equations (29)–(31).

4. Transparency regimes

Let the acoustic wave be clearly transverse, i.e. $\Omega_\parallel = 0$. In this case $\Omega = |\Omega_\perp|$. Introducing a new variable $\theta_\perp = \theta/2 = (1/2)\int_{-\infty}^{\tau} \Omega_\perp \, d\tau'$, from (29) we arrive at the well known theory of the ASIT sine-Gordon equation:

$$\frac{\partial^2 \theta_\perp}{\partial z \, \partial \tau_\perp} = -\frac{\alpha_\perp}{2} \sin \theta_\perp, \tag{32}$$

where $\tau_\perp = t - z/a_\perp$, obtained in [3] for the case of a two-level system.

the one-kink solution of equation (32) has the form

$$\theta_\perp = 4 \arctan \left[\exp\left(\frac{t - z/v}{\tau_p} \right) \right], \tag{33}$$

where the velocity of propagation in the laboratory coordinate system is

$$\frac{1}{v} = \frac{1}{a_\perp} + \frac{\alpha_\perp}{2} \tau_p^2, \tag{34}$$

where τ_p is the duration of the envelope soliton of the transverse component of the form

$$|\Omega_\perp| = 2 \frac{\partial \theta_\perp}{\partial \tau} = \frac{4}{\tau_p} \operatorname{sech}\left(\frac{t - z/v}{\tau_p} \right). \tag{35}$$

Expressions for the populations ρ_{11} and ρ_{33}, derived from (22), (25) and (26) in the common case, are rather large. For its simplification we will consider the temperature T of the paramagnetic crystal to be so low that $T \ll \hbar\omega_0/k_B$, where k_B is the Boltzmann constant. Under $\omega_0 \sim 10^{11}\ \mathrm{s}^{-1}$ we have $T \ll 1$ K. Then we can consider that $W_1 = 1$, $W_2 = W_3 = 0$. In that case from (22), (25) and (26) we find

$$\rho_{11} = \frac{1}{4}\left(1 + \cos^2 \frac{\theta}{2} + 2 \cos \frac{\theta_\parallel}{2} \cos \frac{\theta}{2} + 2 \frac{\Omega_\parallel}{\Omega} \sin \frac{\theta_\parallel}{2} \sin \frac{\theta}{2} + \frac{\Omega_\parallel^2}{\Omega^2} \sin^2 \frac{\theta}{2} \right). \tag{36}$$

The expression for ρ_{33} one can determine from the relation $\rho_{33} = 1 - \rho_{11} - \rho_{22}$. Then from here and also from (28), (36) and (33) we will have

$$\begin{aligned} \rho_{11} &= \tanh^4\left(\frac{t - z/v}{\tau_p} \right), \\ \rho_{22} &= 2 \tanh^2\left(\frac{t - z/v}{\tau_p} \right) \operatorname{sech}^2\left(\frac{t - z/v}{\tau_p} \right), \\ \rho_{33} &= \operatorname{sech}^4\left(\frac{t - z/v}{\tau_p} \right). \end{aligned} \tag{37}$$

We can take into account the longitudinal component, assuming in (31) $\Omega = |\Omega_\perp|$, $\theta = 2\theta_\perp$. Then using (33) and (35), we find

$$\Omega_\parallel = \frac{16\alpha_\parallel}{\alpha_\perp \tau_p^2} \tanh^2\left(\frac{t - z/v}{\tau_p} \right) \operatorname{sech}^2\left(\frac{t - z/v}{\tau_p} \right). \tag{38}$$

However, such consideration of $\Omega_\parallel$ corresponds to the limit $|\Omega_\perp|^2 \gg \Omega_\parallel^2$.

The longitudinal component of the acoustic field gives rise to phase modulation of the transverse component, as follows from (30).

Assuming in the considered limit $\Omega = |\Omega_\perp|$, $\theta = 2\theta_\perp$, from (30), (32) and (34) and taking into account (see (33))

$$\frac{\partial\varphi}{\partial z} = -\left(\frac{1}{v} - \frac{1}{a_\perp}\right)\frac{\partial\varphi}{\partial\tau_\perp} = -\frac{\alpha_\perp}{2}\tau_p^2\frac{\partial\varphi}{\partial\tau_\perp},$$

we will find

$$\delta\omega = -\frac{8\alpha_\parallel}{\alpha_\perp\tau_p^2}\tanh^4\left(\frac{t-z/v}{\tau_p}\right)\operatorname{sech}^2\left(\frac{t-z/v}{\tau_p}\right) \tag{39}$$

for the local frequency shift $\delta\omega = \partial\varphi/\partial\tau_\perp$

Running profiles $|\Omega_\perp|$, $\Omega_\parallel$, $\delta\omega$, and also populations of the quantum levels, corresponding to the limit $|\Omega_\perp|^2 \gg \Omega_\parallel^2$ are presented in figure 2. One can see that, in the process of pulse propagation, spins from the ground level go first to the middle, and then to the third level. The state of the medium, in which all spins are concentrated on the topmost level, corresponds to the central part of the acoustic pulse, where both its components are maximum in value. After propagation of the pulse spins return to the ground state with use of the cascade transition $3 \to 2 \to 1$. The decrease in propagation velocity of SIT and ASIT is usually described just by the processes of excitation and subsequent deexcitation of the medium [1, 2]. The local frequency of the transverse component, according to (39), near the front and back edges feels a shift (figure 2). It could be explained in the following manner. From (15) and (38) one can see that the longitudinal component displaces the middle level downwards, increasing and decreasing the effective quantum frequencies ω_{ef} of the transitions $1 \leftrightarrow 2$ and $2 \leftrightarrow 3$, respectively. At the front and rear edges of the pulse, as has been mentioned earlier, the transition $1 \leftrightarrow 2$ is present. The transverse component decreases its local frequency near both edges, tending to stay in resonance with the transition $1 \leftrightarrow 2$ according to the principle of Le Chatelier and Brown [24]. In the centre, where the transition $2 \leftrightarrow 3$ dominates the longitudinal component is equal to zero. The local frequency of the transverse component, in accordance with the previous reasoning, decreases again to the initial value ω_0.

Let us now increase the longitudinal component so that the condition $\Omega_\parallel^2 \gg |\Omega_\perp|^2$ is achieved. Then $\theta \approx \theta_\parallel$. Let us also say $a_\parallel = a_\perp$. The latter restriction is rather artificial (in solids $a_\parallel > a_\perp$), but it allows us to substantially simplify the mathematical calculations in the following. We will show at the end of the present section that, using typical parameters for the medium and acoustic pulses used in the experiments, the given restriction is not very substantial from the physics point of view. In that case (29) and (31) will be rewritten in the form

$$\frac{\partial|\Omega_\perp|}{\partial z} = -\alpha_\perp\frac{|\Omega_\perp|}{2\Omega}\sin\theta_\parallel, \qquad \frac{\partial\Omega_\parallel}{\partial z} = -\alpha_\parallel\frac{|\Omega_\perp|^2}{2\Omega}\sin\theta_\parallel. \tag{40}$$

From here after multiplication of the first equation by $2|\Omega_\perp|$ and integration with consideration of the zero values of $2|\Omega_\perp|$ and $\Omega_\parallel$ at infinity we will arrive at

$$\Omega_\parallel = \frac{\alpha_\parallel}{\alpha_\perp}|\Omega_\perp|^2. \tag{41}$$

Note that a similar relation between $\Omega_\parallel$ and $|\Omega_\perp|^2$ was found in [6] by solving the problem of longitudinal–transverse acoustic pulses propagating in the system of paramagnetic impurities with effective spin $S = 1/2$.

From (40) and (41), taking into account the limit concerned is $\Omega \approx \Omega_\parallel$, we come again to the sine-Gordon equation:

$$\frac{\partial^2\theta_\parallel}{\partial z\,\partial\tau} = -\alpha_\perp\sin\theta_\parallel. \tag{42}$$

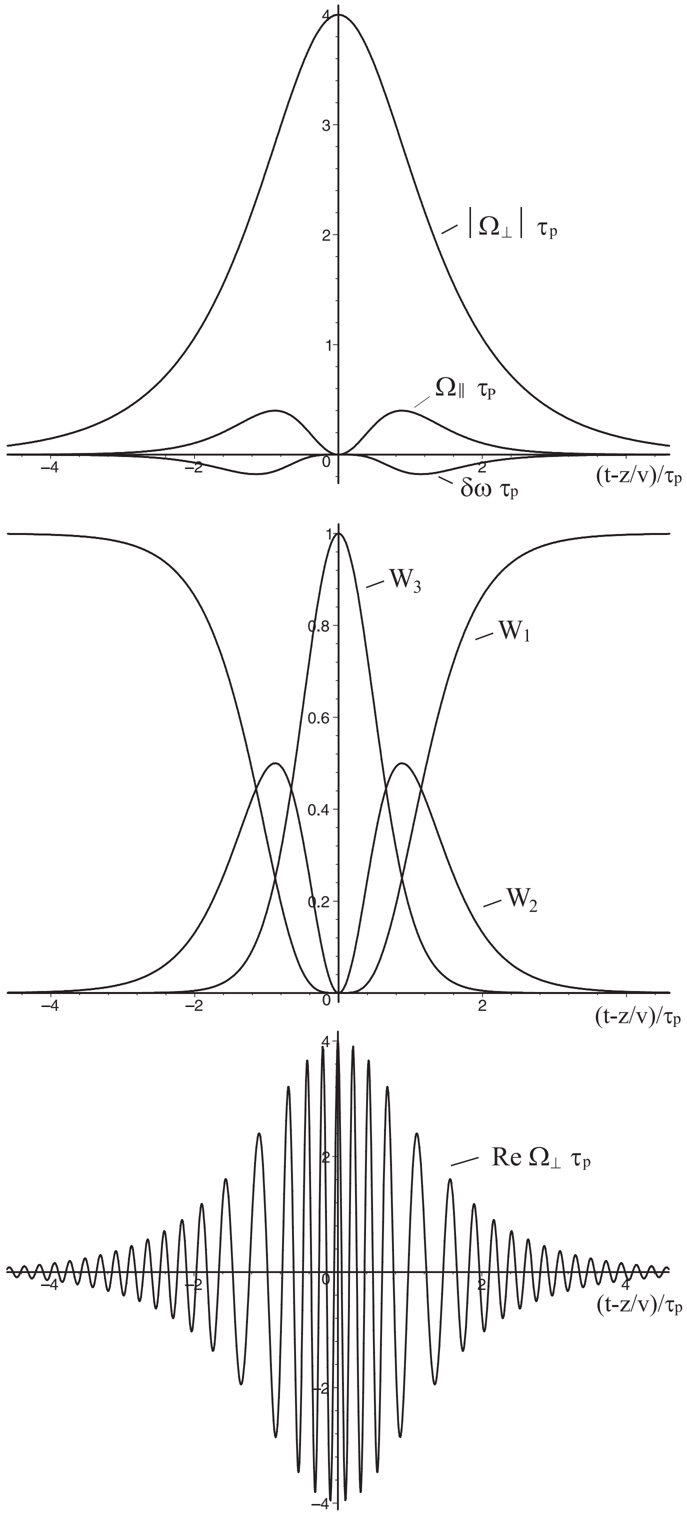

Figure 2. Schematic view of the running dimensionless profiles $|\Omega_\perp|$, $\Omega_\parallel$, $\delta\omega$, population of the quantum levels and $\mathrm{Re}\,\Omega_\perp$ in the case of $|\Omega_\perp|^2 \gg \Omega_\parallel^2$ (the ASIT regime).

This equation is the same as (32) given the substitutions $\theta_\perp \to \theta_\parallel$ and $\alpha_\perp/2 \to \alpha_\parallel$.

From here and from (41) we will find the corresponding one-soliton equations:

$$\theta_\parallel = 4\arctan\left[\exp\left(\frac{t - z/v}{\tau_p}\right)\right], \tag{43}$$

$$\Omega_\parallel = \frac{2}{\tau_p}\,\mathrm{sech}\left(\frac{t - z/v}{\tau_p}\right), \tag{44}$$

$$|\Omega_\perp| = \sqrt{\frac{2\alpha_\perp}{\alpha_\parallel \tau_p}}\,\mathrm{sech}^{1/2}\left(\frac{t - z/v}{\tau_p}\right), \tag{45}$$

$$\frac{1}{v} = \frac{1}{a_\perp} + \alpha_\perp \tau_p^2. \tag{46}$$

Using (28), we find

$$\delta\omega = \frac{\partial\varphi}{\partial\tau} = -\frac{1}{\tau_p}\,\mathrm{sech}\left(\frac{t - z/v}{\tau_p}\right). \tag{47}$$

From this one can see that, in the limit of the large longitudinal component, the local frequency of the transverse component experiences a negative shift in the centre of the pulse. To understand this given situation, it is necessary to analyse the behaviour of the quantum level populations. Assuming in (36) $\Omega = \Omega_\parallel$, $\theta = \theta_\parallel$, we find $\rho_{11} \approx 1$. From (28) it also follows that changes in ρ_{22} are vanishingly small: $\rho_{22} \sim (|\Omega_\perp|/\Omega_\parallel)^2 \ll 1$ in the limit concerned. So, the populations of quantum levels in practice do not suffer dynamics during propagation of the soliton of the form (43) and (45). In our case ($T \ll \hbar\omega_0/k_B$) all spins in practice stay in the ground state. This happens because the large longitudinal component eliminates the transverse component from resonance with the medium. It is evident from (44) and (15) that the transition $1 \leftrightarrow 2$ experiences a negative frequency shift. According to this, phase modulation of the transverse component brings about a decrease of the local carrier frequency in the centre.

In spite of the capture of the quantum level populations, the given regime of acoustic transparency is accompanied by a decrease in the propagation velocity even greater than in the case of the large resonant transverse component, when the regime of simple ASIT is realized (see (34) and (46)). Here an analogy to the electromagnetically induced transparency effect arises. In the latter case the three-level system of λ transitions becomes transparent for the transition $1 \leftrightarrow 3$ in the presence of the power pumping resonance to the transition $2 \leftrightarrow 3$. It is also accompanied by the determination of populations and group velocity upon the substantial decrease in the signal [25, 26].

In our case the capture of populations is reached by the video pulse of a longitudinal wave. The decrease in the propagation velocity, according to (46), depends only on the parameters of the interaction of the transverse wave and spins. Therefore we term the concerned effect the acoustic longitudinal–transverse transparency (ALTT).

Here one can understand the mechanism of the slowing down of propagation on the basis of the dispersion law analysis. Thus, from (27) and (28) one can see that at $\Omega_\parallel^2 \gg |\Omega_\perp|^2$ non-diagonal elements $\hat{R}$, forming the dispersion law, are linear on small deviations $|\Omega_\perp|/\Omega_\parallel$, and not quadratic, at vanishingly small changes in the population. So, the influence of the paramagnetic impurities on the velocity is determined by diagonal elements of $\hat{R}$. Assuming in (42) $\theta_\parallel \sim \exp(\mathrm{i}\delta\omega t - \mathrm{i}\delta k t)$, after linearization $\sin\theta_\parallel \approx \theta_\parallel$ we find the dispersion equation $\delta k = \delta\omega/a_\perp - \alpha_\perp/\delta\omega$. Here $\delta\omega$ and δk have the corresponding meanings of spectral shifts from the carrier frequency ω_0 and wavenumber k. From here we find $1/v = \mathrm{d}\delta k/\mathrm{d}\delta\omega = 1/a_\perp + \alpha_\perp/(\delta\omega)^2$ for the group velocity. Taking $(\delta\omega)^2 \sim 1/\tau_p^2$ (see (46)), we arrive at the relation (46).

In figure 3 profiles of the longitudinal $\Omega_{\|}$ and transverse $|\Omega_{\perp}|$ components, changes of the local frequency $\delta\omega$ for the latter and populations of the quantum spin levels are shown. From the comparison of figures 2 and 3 one can see the principal differences between the ASIT and ALTT regimes. One of the main differences consists in the dynamics of the populations of spin quantum levels: in the case of ASIT a full inversion of populations with a subsequent return to the initial state takes place, while during the ALTT regime the populations stay practically unchanged. The consequence of this difference is the different character of the phase modulation of the transverse component during ASIT and ALTT (see figures 2 and 3).

Using (44) and (45) we rewrite the condition $\Omega_{\|}^2 \gg |\Omega_{\perp}|^2$, at which the ALTT regime can be realized, in the form

$$1 \ll \omega_0 \tau_{\mathrm{p}} \ll (3G_{11}/2G_{44})^2. \tag{48}$$

The left-hand side of this double inequality corresponds to the criterion of quasi-monochromaticity of the transverse component, and the right-hand side is the condition of large longitudinal component.

To satisfy both parts of inequality (48) then $G_{11}/G_{44} \sim 10$. The search for materials with such properties is not a very simple problem. Therefore it is necessary to note the works [6, 7] where the problem of ASIT for longitudinal–transverse waves in the system of spins $S = 1/2$ was solved without SVEA for the transverse component. In this case the accomplishment of the quasi-monochromaticity condition is not necessary, but at the same time we have to mention the resonant interaction of the pulse with the medium. So, the model of the system of spins $S = 1/2$ cannot be fully adequate to represent the real situation.

Note that values of the relative deformation $\mathcal{E}_{\|}$ and $|\mathcal{E}_{\perp}|$ during ALTT are large in value. The inequality $\Omega_{\|}^2 \gg |\Omega_{\perp}|^2$ will be held to be true due to $G_{11}^2 \gg G_{44}^2$.

Equation (42) and expressions (41), (43)–(47) are obtained by us for the case of the equality of velocities of transverse $a_{\perp}$ and longitudinal $a_{\|}$ sound. Usually $a_{\|} > a_{\perp}$ is in the solid state [14, 27]. One can obtain the stationary solution, corresponding to the running waves, in this case too. In fact, assuming that dynamical parameters in (29)–(31) depend on z and τ in the form $\xi \equiv \tau - (1/v - 1/a_{\|})z$, we will find the system of ordinary integro-differential equations

$$|\Omega_{\perp}|' = \gamma_{\perp} \frac{|\Omega_{\perp}|}{\Omega} \cos\frac{\theta_{\|}}{2} \sin\frac{\theta}{2}, \tag{49}$$

$$\varphi' = -\sigma_{\perp} \frac{\Omega_{\|}}{\Omega^2} \sin^2\frac{\theta}{2}, \tag{50}$$

$$\Omega_{\|}' = \gamma_{\|} \frac{|\Omega_{\perp}|^2}{\Omega} \sin\theta, \tag{51}$$

where $\gamma_{\perp} = \alpha_{\perp}/(1/v - 1/a_{\perp})$, $\gamma_{\|} = \alpha_{\|}/(1/v - 1/a_{\|})$ and primes mark the derivatives with respect to ξ.

In the limit of ALTT ($\theta \approx \theta_{\|}$) from (49) and (51) we will arrive at an expression of the form (41), taking into account the substitution

$$\frac{\alpha_{\|}}{\alpha_{\perp}} \to \frac{\alpha_{\|}}{\alpha_{\perp}} \frac{1/v - 1/a_{\perp}}{1/v - 1/a_{\|}},$$

and also the equation

$$\theta_{\|}'' = \frac{1}{\tau_{\mathrm{p}}^2} \sin\theta_{\|},$$

where $1/\tau_{\mathrm{p}}^2 = \alpha_{\perp}/(1/v - 1/a_{\perp})$.

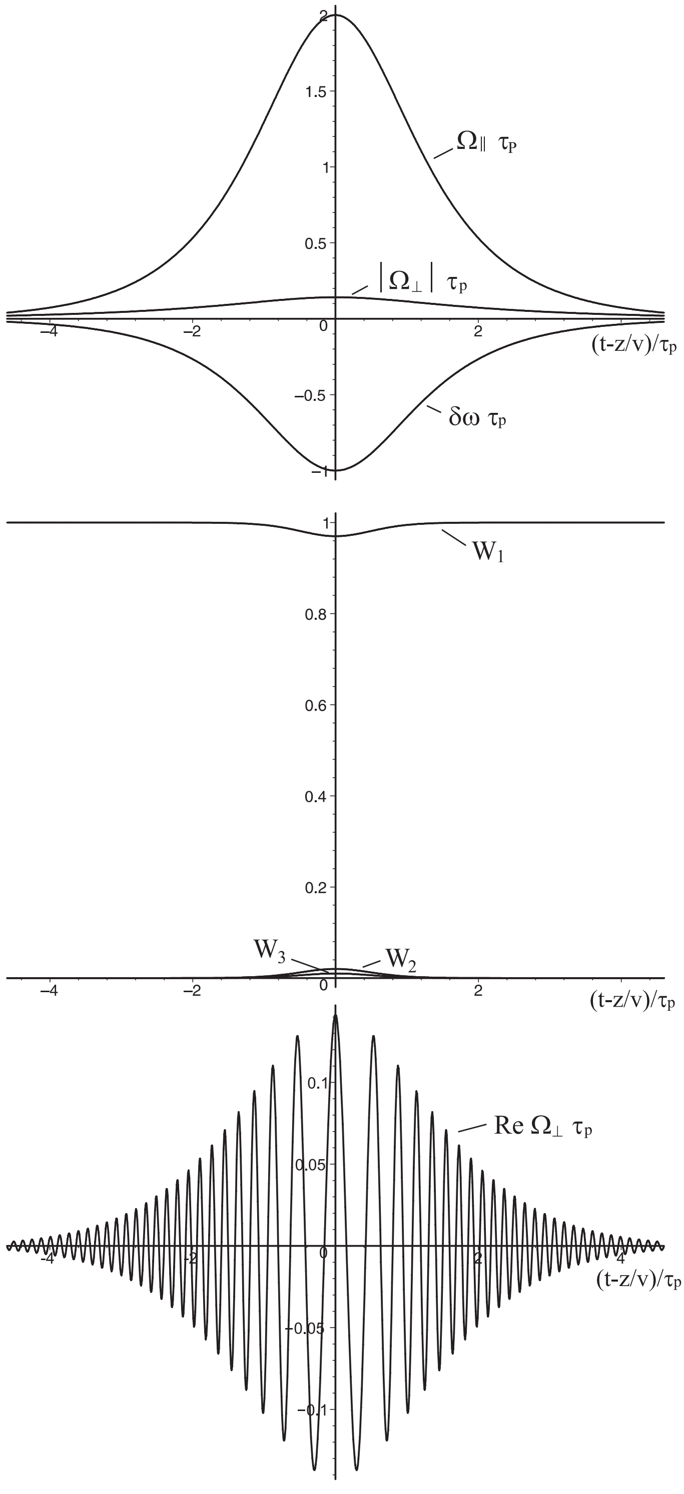

Figure 3. Schematic view of the running dimensionless profiles $|\Omega_\perp|$, $\Omega_\parallel$, $\delta\omega$, population of the quantum levels and $\mathrm{Re}\,\Omega_\perp$ in the case of $|\Omega_\perp|^2 \gg \Omega_\parallel^2$ (the ALTT regime).

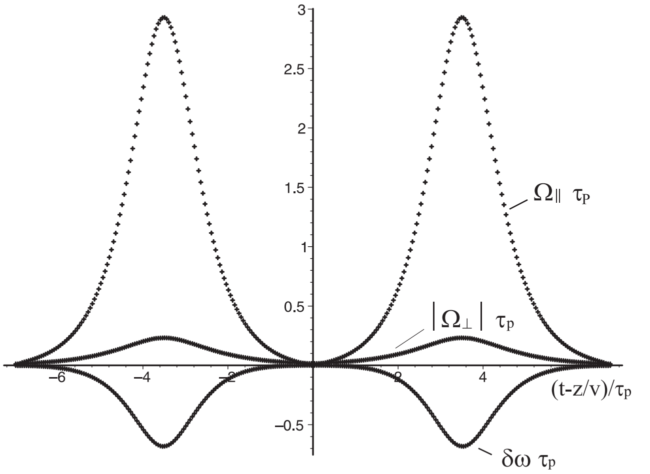

Figure 4. Results of the numerical analysis of system (49)–(51).

The solution of the given equation, corresponding to the solitary longitudinal–transverse pulse, is given by (43). From here one can see that the expression (46) is correct in the presence of detuning between $a_\parallel$ and $a_\perp$. Expressions (44) and (47) also do not change their form. At the same time, in (45) it is necessary to make a substitution

$$\frac{\alpha_\perp}{\alpha_\parallel} \to \frac{\alpha_\perp}{\alpha_\parallel}\left[1 + \frac{1}{\alpha_\perp \tau_\mathrm{p}^2}\left(\frac{1}{a_\perp} - \frac{1}{a_\parallel}\right)\right]. \tag{52}$$

From the comparison of (44) and (45), taking into account the given substitution, we make a derivation about the relative decrease of the role of the transverse component video pulse in the presence of a velocity shift, as $1/a_\perp > 1/a_\parallel$. Because of the given shift the effectiveness of the energy pumping from the quasi-monochromatic transverse component into the video pulse of the longitudinal wave decreases. One could also mention the violation of the condition of long–short wave resonance of Zakharov–Benney type. Taking for Fe^{2+}:MgO [10, 28] $n \sim 10^{17}$ cm^{-1}, $\omega_0 \sim 10^{11}$ s^{-1}, $G_{44} \sim 10^{-13}$ erg, $\rho \simeq 2$ g cm^{-3}, $a_\perp \simeq 5 \times 10^5$ cm s^{-1}, $a_\parallel \simeq 10^6$ cm s^{-1} and $\tau_\mathrm{p} \sim 10^{-8}$ s, we find $(1/a_\perp - 1/a_\parallel)/(\alpha_\perp \tau_\mathrm{p}^2) \sim 10^{-2} \ll 1$ (see (52)). Thus, typical shifts of $a_\perp$ and $a_\parallel$ cannot, in principle, influence the observation of ALTT under experimental conditions.

From the previous analysis one can see that the ASIT regime changes to ALTT on the continuous increase of the longitudinal component with respect to the transverse.

For clarification of the nature of the conversion from ASIT to ALTT the system (49)–(51), in which an assumption is made that stationary running pulses are formed in the medium, was subjected to numerical analysis. The corresponding results are displayed in figure 4. As far as the specific gravity of the two-peaked longitudinal component is increased a small hole appears in the centre of the transverse component. With the further increase of $\Omega_\parallel$ the hole becomes deeper and two maxima appear in the profile of $|\Omega_\perp|$, moving away from each other. During $\Omega_\parallel^2 \gg |\Omega_\perp|^2$ the maxima disperse so far that they can be considered individually as solitary pulses. As far as peaks in the profiles of $|\Omega_\perp|$ and $\Omega_\parallel$ disperse equivalently happens with peaks in the $\delta\omega$ profile: each of the peaks correlates with the corresponding peaks in the transverse and longitudinal components. In this way the phase modulated pulse of ASIT (figure 2) shades into the ALTT pulse (figure 3). The same reasoning is also used for the dynamics of the ground spin state population. Note that the results of numerical simulations are in good agreement with analytical solutions, which were obtained here for two opposite cases, corresponding to ASIT and ALTT.

5. Conclusion

In the present work an investigation is conducted to show that consideration of the two-component (longitudinal–transverse) structure of the acoustic pulse results in a deep understanding of resonant transparency mechanisms in the system of paramagnetic impurities. In this case the roles of both components are strongly varied: the transverse component induces quantum transitions between resonant Zeeman sublevels, while the longitudinal component changes the frequencies of the given transitions in a dynamical way. In just such a geometry (Faraday geometry) for two opposite limits we predict two soliton regimes for the propagation of acoustic pulses, corresponding to ASIT and ALTT. In order of magnitude the decrease in the propagation velocity of pulses in both limits is equal. However, the behaviour of the medium is different: in the first case the full inversion of quantum level populations happens, while in the second case the populations practically do not change.

During propagation of the acoustic pulse under an arbitrary angle the longitudinal component can excite a transition $1 \leftrightarrow 3$ [2]. Just as in the case of ASIT, when a particular longitudinal pulse propagated at an angle of 45° to the direction of $\mathbf{B_0}$, calling it a $1 \leftrightarrow 3$ transition was considered in [2]. So the investigation of longitudinal–transverse resonant pulse propagation under arbitrary angles with regard to $\mathbf{B_0}$ was interesting. It was not excepted that in the process new nonlinear mechanisms for resonant acoustic transparency could be realized.

Usually in solids the quantum transitions between Zeeman sublevels are subjected to strong inhomogeneous broadening, which, generally speaking, it is necessary to consider. For two-component pulses the consideration given seems not so simple, as in the case of one-component (longitudinal [2] or transverse [3]) pulses. However, later we are planning to work on solving this problem and the problem of the area theorem for longitudinal–transverse acoustic pulses.

Acknowledgment

This work was carried out with the financial support of the Russian Foundation of Basic Research (project: 02-02-17710a).

References

[1] McCall S L and Hahn E L 1967 *Phys. Rev. Lett.* **18** 908
[2] Shiren N S 1970 *Phys. Rev.* B **2** 2471
[3] Denisenko G A 1971 *Zh. Eksp. Teor. Fiz.* **60** 2269
Denisenko G A 1971 *Sov. Phys.—JETP* **33** 1220 (Engl. Transl.)
[4] Samartsev V V, Smolyakov B P and Sharipov R Z 1974 *Pis. Zh. Eksp. Teor. Fiz.* **20** 644
Samartsev V V, Smolyakov B P and Sharipov R Z 1974 *JETP Lett.* **20** 296 (Engl. Transl.)
[5] Voronkov S V and Sazonov S V 2001 *Zh. Eksp. Teor. Fiz.* **120** 269
Voronkov S V and Sazonov S V 2001 *JETP* **93** 236 (Engl. Transl.)
[6] Zabolotskii A A 2003 *Zh. Eksp. Teor. Fiz.* **123** 560
Zabolotskii A A 2003 *JETP* **96** 496 (Engl. Transl.)
[7] Zabolotskii A A 2002 *Pis. Zh. Eksp. Teor. Fiz.* **76** 709
Zabolotskii A A 2002 *JETP Lett.* **76** 607 (Engl. Transl.)
[8] Sazonov S V 1994 *J. Phys.: Condens. Matter* **6** 6295
[9] Tuckler J W and Pampton V W 1972 *Microwave Ultrasonics in Solid State Physics* (Amsterdam: North-Holland)
[10] Sazonov S V 2000 *Zh. Eksp. Teor. Fiz.* **118** 20

Sazonov S V 2000 *JETP* **91** 16 (Engl. Transl.)

[11] Altshuler S A and Kozyrev B M 1972 *Electron Paramagnetic Resonance in Compounds of Transition Elements* (Moscow: Nauka) (1997, (New York: Halsted))

[12] Golenishchev-Kutuzov V A, Samartsev V V, Solovarov N K and Khabibulin B M 1977 *Magnetic Quantum Acoustics* (Moscow: Nauka)

[13] Kopvillem U Kh, Samartsev V V and Solovarov N K 1976 *Adv. Mol. Relax. Process.* **8** 241

[14] Landau L D and Lifshitz E M 1987 *Theory of Elasticity* (*Course of Theoretical Physics* vol 7) (Moscow: Nauka) (1986 (New York: Pergamon))

[15] Eilbeck J C, Gibbon J D, Caudrey P J and Bullough R K 1973 *J. Phys. A: Math. Gen.* **6** 1337

[16] Caudrey P J, Eilbeck J C, Gibbon J D and Bullough R K 1973 *J. Phys. A: Math. Gen.* **6** L53

[17] Vinogradova M B, Rudenko O V and Sukhorukov A P 1990 *The Theory of Waves* (Moscow: Nauka)

[18] Moiseev N N 1981 *Asymptotic Methods of Nonlinear Mechanics* (Moscow: Nauka)

[19] Nayfeh A H 1981 *Introduction to Perturbation Techniques* (New York: Wiley)

[20] Gantmacher F R 1966 *The Theory of Marices* (Moscow: Nauka) (1959 (New York: Chelsea))

[21] Dodd R K, Eilbeck J C, Gibbon J D and Morris H C 1982 *Solitons and Nonlinear Waves Equations* (New York: Academic)

[22] Zakharov V E 1972 *Zh. Eksp. Teor. Fiz.* **62** 1745
Zakharov V E 1972 *Sov. Phys.—JETP* **35** 908 (Engl. Transl.)

[23] Yajima N and Oikawa M 1976 *Prog. Theor. Phys.* **56** 1719

[24] Kubo R 1968 *Thermodynamics* (Amsterdam: North-Holland)

[25] Boller K J, Imamoglu A and Harris S E 1991 *Phys. Rev. Lett.* **66** 2593

[26] Harris S E 1997 *Phys. Today* **50** (7) 36

[27] Anderson O 1965 *Physical Acoustics: Principles and Methods* part B *Lattice Dynamics* vol 3, ed W P Mason (New York: Academic)

[28] Sazonov S V 2002 *Pis. Zh. Eksp. Teor. Fiz.* **76** 176
Sazonov S V 2002 *JETP Lett.* **76** 143 (Engl. Transl.)